Large Extra Dimensions, Sterile neutrinos and Solar Neutrino Data

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Solar, atmospheric and LSND neutrino oscillation results require a light sterile neutrino, ν_B, which can exist in the bulk of extra dimensions. Solar ν_e, confined to the brane, can oscillate in the vacuum to the zero mode of ν_B and via successive MSW transitions to Kaluza-Klein states of ν_B. This new way to fit solar data is provided by both low and intermediate string scale models. From average rates seen in the three types of solar experiments, the Super-Kamiokande spectrum is predicted with 73% probability, but dips characteristic of the 0.06 mm extra dimension should be seen in the SNO spectrum.

The four-neutrino scheme in which the solar ν_e deficit is explained by $\nu_e \to \nu_s$ (where ν_s is a sterile neutrino), the atmospheric ν_μ/ν_e anomaly is attributed to $\nu_\mu \to \nu_\tau$, and the heavier ν_μ and ν_τ share the role of hot dark matter was originally proposed [1] in order to explain those three phenomena. Later the LSND experiment [2], which observed $\bar{\nu}_\mu \to \bar{\nu}_e$, provided a measure of the mass difference between the nearly degenerate $\nu_e - \nu_s$ and $\nu_\mu - \nu_\tau$ pairs and required the three mass differences that were already present in that neutrino scheme. Exactly this same pattern of neutrino masses and mixings appears necessary to allow production of heavy elements (A $\gtrsim$ 100) by type II supernovae [3]. While qualitatively this neutrino scheme seems to explain all existing neutrino phenomena, solar neutrino observations are now sufficiently constraining that the small-angle MSW $\nu_e \to \nu_s$ explanation appears to be in some difficulty [4]. Although providing better fits to the solar data, even active-active transitions in a three-neutrino scheme do not give a quantitatively good explanation of those data. In this Letter we point out that there is a way to achieve an excellent fit and rescue the apparently needed four-neutrino scheme using large extra dimensions. This is motivated by the latest developments in string theories, which have made plausible the interesting possibility that such large extra space dimensions ($\lesssim$mm) [5] can exist in conjunction with low or intermediate fundamental string scales, M_*. Using such models and the average rates from the three types of solar experiments, we predict the energy spectra from the Super-Kamiokande [4] and SNO experiments employing a combination of vacuum and MSW oscillations. The neutrino-electron scattering results from Super-Kamiokande is predicted with a χ^2 corresponding to 73% probability, while the better energy resolution of the charged-current interactions from SNO should show the predicted dips in the spectrum.

We have found two classes of models giving the desired phenomenology (see [6] for details). The first model has low M_*, as has been used to give an alternative to the conventional high-scale SUSY GUT theories in solving the gauge hierarchy problem and providing low energy signals of string theories. Since there are no large scales to implement the conventional seesaw mechanism, a way to get small neutrino masses is to include singlet neutrinos in the bulk, in combination with the assumption [7] of an effective global $B-L$ symmetry in the theory below M_*. The large size of the bulk suppresses neutrino masses to the desired level. The low Kaluza-Klein ("KK") excitations have a mass $\sim R^{-1} \sim 10^{-3}$ eV, which not only provide a natural way to understand the lightness of the sterile neutrino [8], but also give extra sterile neutrino states in the mass range that can influence the shape of the solar neutrino spectrum [9].

This model has one bulk neutrino, $\nu_B(x,y)$, y being the coordinate of the fifth dimension, and the standard model on the brand. The $\nu_B(x,y)$ is assumed to couple to the lepton doublet of the standard model L and of course have a five-dimensional kinetic energy term. After electroweak symmetry breaking at scale v_{wk}, the $\nu_e - \nu_{B,R}$ coupling leads to $m_\nu = h\frac{M_* v_{wk}}{M_{P\ell}} \sim 10^{-5}$ eV, where $M_{P\ell}$ is the Planck mass and h is the Yukawa coupling. Note that this suppression is independent of the number and radius hierarchy of the extra dimensions, provided that ν_B propagates in the whole bulk. Even if the bulk is six or higher dimensional, there is only one mm-scale dimension. The smaller dimensions will contribute only to the relationship between $M_{P\ell}$ and M_*, but their KK excitations will be very heavy and decouple from the neutrino spectrum, making the analysis as in five dimensions.

The direct Dirac or Majorana mass terms for the bulk neutrino can be forbidden by an appropriate choice of geometry and dimension of the bulk in which the ν_B resides, making an ultralight ν_s natural. For instance, in 5 dimensions the Z_2 orbifold symmetry under which $y \to -y$ combined with $B-L$ symmetry guarantee this for the conventional definition of charge conjugation.

In order to fit neutrino data, one needs to include new physics in the brane that will generate a Majorana mass matrix for the three standard model neutrinos of the form

δ_{ab} (where $a, b = e, \mu, \tau$). For $\delta_{\mu\tau}$ much bigger than the other elements, the $\nu_{\mu,\tau}$ in effect decouple from the $\nu_{e,s}$ and do not affect the mixing between the bulk neutrino modes and the ν_e. Further, this leads to maximal mixing in the $\mu - \tau$ sector, as is needed to understand the atmospheric neutrino data. If $\delta_{\mu\tau} \sim$eV, then this provides an explanation of the LSND observations. One way to generate this pattern is to consider an $L_e + L_\mu - L_\tau$ symmetric extension of the standard model with additional doubly-charged singlet ($Y = 4$) and $Y = 2$ triplet scalar fields. In the rest of the Letter, we focus only on the $\nu_e - \nu_s$ sector and how we fit the solar neutrino data.

The mass matrix for the ν_e , ν_s sector can be written

$$(\nu_e \nu_{0B} \nu'_{B,-} \nu'_{B,+}) \begin{pmatrix} \delta_{ee} & m & \sqrt{2}m & 0 \\ m & 0 & 0 & 0 \\ \sqrt{2}m & 0 & 0 & \partial_5 \\ 0 & 0 & \partial_5 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_{0B} \\ \nu'_{B,-} \\ \nu'_{B,+} \end{pmatrix}, \quad (1)$$

where ν'_B represents the KK excitations, and the off-diagonal term $\sqrt{2}m$ is actually an infinite row vector of the form $\sqrt{2}m(1, 1, \cdots)$. The operator ∂_5 stands for the diagonal and infinite KK mass matrix whose n-th entry is given by n/R. Using this short-hand notation makes it easier to calculate the exact eigenvalues m_n and the eigenstates of this mass matrix,

$$m_n = \delta_{ee} + \pi m^2 R \cot(\pi m_n R). \quad (2)$$

The equation for eigenstates is

$$N_n \tilde{\nu}_n = \nu_e + \frac{m}{m_n} \nu_{0B} + \frac{\sqrt{2}m \left(m_n \nu'_{B,-} + \partial_5 \nu'_{B,+} \right)}{m_n^2 - \partial_5^2}, \quad (3)$$

where the sum over the KK modes in the last term is implicit. N_n is the normalization factor given by

$$N_n^2 = 1 + m^2 \pi^2 R^2 + \frac{(m_n - \delta_{ee})^2}{m^2}. \quad (4)$$

Note that in the limit of $\delta_{ee} = 0$, the ν_e and $\nu_{B,0}$ are two, two-component spinors that form a Dirac fermion with mass m. The KK modes come in pairs of mass $m_n = \pm k_n/R$, with k_n a positive integer, and they couple to the ν_e approximately as mR. Once we include the effect of $\delta_{ee} \neq 0$, they become Majorana fermions with masses given by $m_1 \approx +\delta_{ee}/2 + m$ and $m_2 \approx +\delta_{ee}/2 - m$, and they are maximally mixed; i.e., the two mass eigenstates are $\nu_{1,2} \simeq \frac{\nu_e \pm \nu_{B,0}}{\sqrt{2}}$. Thus as the ν_e produced in a weak interaction process evolves, it oscillates to the $\nu_{B,0}$ state with an oscillation length $D \simeq E/(2m\delta_{ee})$, which for natural values of m, δ_{ee} gives D of order of the Sun-Earth distance so that our model leads to vacuum oscillation ("VO") of the solar neutrinos. Furthermore, since the ν_e also mixes with the KK modes of the bulk neutrinos with a $\delta m^2 \sim 10^{-5}$ eV^2, this brings in the MSW resonance transition of ν_e to $\nu_{B,KK}$ modes at higher energies.

The second way to achieve the same phenomenology is to use a much higher string scale associated with local $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry of the left-right symmetric model in the brane, coupled to a single ν_B in the large fifth dimensional bulk. Using the usual fermion content of the left-right models and breaking the right-handed symmetry by Higgs doublets, one obtains the see-saw type matrix in the presence of the infinite tower of KK modes. The heavy right-handed neutrino now decouples, leading to the following neutrino mass matrix in the one generation case (the basis is $(\nu_e \nu_{0B} \nu'_{B,-} \nu'_{B,+})$)

$$\frac{1}{M} \begin{pmatrix} m^2 + \Delta & m\alpha & \sqrt{2}m\alpha & 0 \\ m\alpha & \alpha^2 & \sqrt{2}\alpha^2 & 0 \\ \sqrt{2}m\alpha & \sqrt{2}\alpha^2 & 0 & \partial_5 \\ 0 & 0 & \partial_5 & 0 \end{pmatrix}, \quad (5)$$

where $\alpha \simeq \frac{h_\ell M_* v_R}{M_{P\ell}}$, $m = h_\ell v_{wk}$, $M = v_R^2/M_*$, and v_R is the scale of $SU(2)_R$ breaking. Δ denotes the radiative correction induced due to the extrapolation from the v_R scale to the weak scale. The lowest eigenvalue of this mass matrix is $\sim \frac{\alpha^2 \Delta}{M(\alpha^2 + m^2)}$, and the next lowest one is $\frac{m^2 + \alpha^2}{M}$. For $m, \alpha \sim 1 - 5$ MeV (similar to the first generation fermion mass) and $M \simeq 10^9$ GeV, we get this eigenvalue to be of order $10^{-6} - 2.5 \times 10^{-5}$ eV. Its square is therefore in the range where the VO solution to the solar neutrino puzzle can be applied. Also for $m \simeq \alpha$, the mixing angle between the zero eigenvalue mode and this mode is maximal. Thus this model has properties similar to the first model for neutrinos, and below we carry out our fit to solar neutrino data using the latter.

For propagation in solar matter, the eigenvectors and eigenvalues can be found by replacing the squared mass matrix, M^2, for the neutrinos, with $M^2 + H$, where $H = 2E\rho_e$ when acting on ν_e (where $\rho_e = \sqrt{2} G_F (n_e - n_n/2)$), and zero on sterile neutrinos. Defining

$$w = \frac{E\rho_e}{m_n \delta_{ee}} + \sqrt{1 + \left(\frac{E\rho_e}{m_n \delta_{ee}} \right)^2}, \quad (6)$$

($w = 1$ in vacuum) Eq. 2 becomes

$$m_n = w\delta_{ee} + \pi m^2 R \cot(\pi m_n R). \quad (7)$$

The eigenvectors are as in Eq. 3, except the third term acquires an additional factor $1/w$.

In fitting the solar data using VO, the strategy generally employed is to suppress ^{7}Be neutrinos while also reducing the ^{8}B neutrinos by more than 50%. The water data then requires an additional contribution, which, in the case of active VO, is provided by the neutral current cross section amounting to about 16% of the charged current one. Thus in a pure two-neutrino oscillation picture, VO does not work for active to sterile oscillation. In our model, however, both vacuum oscillations and MSW oscillations are important, since the lowest mass pair of neutrinos is split by a very small mass difference, whereas the KK states have to be separated by $> 10^{-3}$ eV because of the limits from gravity experiments. We can use the

first node of the survival probability, P_{ee}, to suppress the ^{7}Be. Going up in energy toward ^{8}B neutrinos, P_{ee}, which in the VO case would have risen to very near one, is reduced by the small-angle MSW transitions to the different KK excitations of the bulk sterile neutrinos, as is clear from Fig. 1. This is a new way to fit the solar neutrino data in models with large extra dimensions.

To do the fit, we studied the time evolution of the ν_e state with a program supplied by W. Haxton [10], but updated to use a recent solar model [11] and modified to do all neutrino transport within the Sun numerically, using no adiabatic approximation. Changes were also necessary for oscillations into sterile neutrinos and to generalize beyond the two-neutrino model, for up to 14 neutrinos contribute for the solutions we considered.

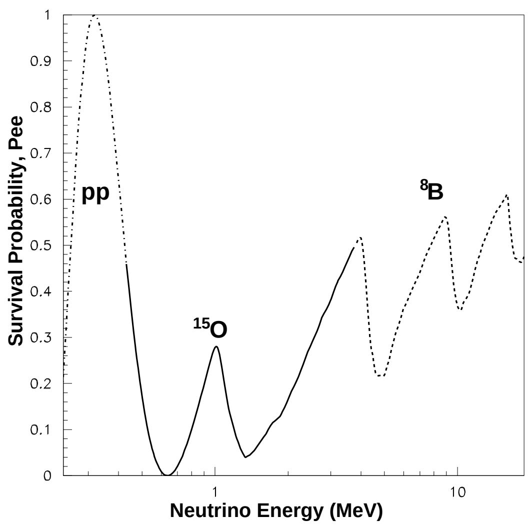

FIG. 1. Energy dependence of the ν_e survival probability when $R = 58\,\mu$m, $mR = 0.0094$, and $\delta_{ee} = 0.84 \times 10^{-7}$ eV. The dot-dashed part of the curve assumes the radial dependence in the Sun for neutrinos from the pp reaction, the solid part assumes ^{15}O radial dependence, and the dashed part assumes ^8B radial dependence.

For comparison with experimental results, tables of detector sensitivity for the Chlorine and Gallium experiments were taken from Bahcall's web site [11]. The Super-Kamiokande detector sensitivity was modeled using [12] for the resolution in the signal from Cherenkov light. More details are given elsewhere [6]. Calculations of P_{ee}, averaged over the response of detectors, were compared with measurements. While theoretical uncertainties in the solar model and detector response were included in the computation of χ^2 as described in Ref. [13], the measurement results given here include only experimental statistical and systematic errors added in quadrature. The Chlorine P_{ee} from Homestake [14] is 0.332 ± 0.030. Gallium results [15] for SAGE, GALLEX and GNO were combined to give a P_{ee} of 0.579 ± 0.039. The $5.0-20$ MeV, 1258-day Super-K experimental P_{ee} [4] is 0.451 ± 0.016. The best fits were with $R \approx 58\,\mu m$, mR around 0.0094, and $\delta_{ee} \sim 0.84 \times 10^{-7}$ eV, corresponding to $\delta m^2 \sim 0.53 \times 10^{-11}$ eV^2, giving average P_{ee} for Chlorine, Gallium, and water of 0.383, 0.533, and 0.450, respectively, and the P_{ee} energy dependence shown in Fig. 1. For two-neutrino oscillations, the mixing angle is $sin^2 2\theta$, whereas here the coupling between ν_e and the first KK excitation replaces $sin^2 2\theta$ by $4m^2R^2 = 0.00035$.

Vacuum oscillations between the lowest two mass eigenstates nearly eliminate electron neutrinos with energies of 0.63 MeV/(2n+1) for n = 0, 1, 2, ... Thus Fig. 1 shows nearly zero P_{ee} near 0.63 MeV, partly eliminating the ^7Be contribution at 0.862 MeV, and giving a dip at the lowest neutrino energy. MSW resonances with mass pairs of higher KK states start causing the third and fourth eigenstates to be significantly occupied above $\sim$ 0.8 MeV, the fifth and sixth eigenstates above $\sim$ 3.7 MeV, the 7'th and 8'th above $\sim$ 8.6 MeV, and the 9'th and 10'th above $\sim$ 15.2 MeV. Fig. 1 shows dips in P_{ee} just above these energy thresholds.

The expected energy dependence of P_{ee} is compared with Super-K data [4] in Fig. 2. The uncertainties are statistical only. The parameters used in making Fig. 2 were chosen to provide a good fit to the total rates only; they were not adjusted to fit this spectrum. Combining spectrum data with rates using the method described in Ref. [16] gives $\chi^2 = 14.0$ for the spectrum predicted from the fit to total rates. With 18 degrees of freedom, the probability of $\chi^2 > 14.0$ is 73%. A fit with δ_{ee} constrained to be very small to eliminate vacuum oscillations increased the best fit χ^2 from 3.4 to 4.4. The same parameters then used with the Super-K spectrum gave $\chi^2 = 18.7$ (probability 41%). This is comparable to $\chi^2 = 19.0$ for an energy independent spectrum.

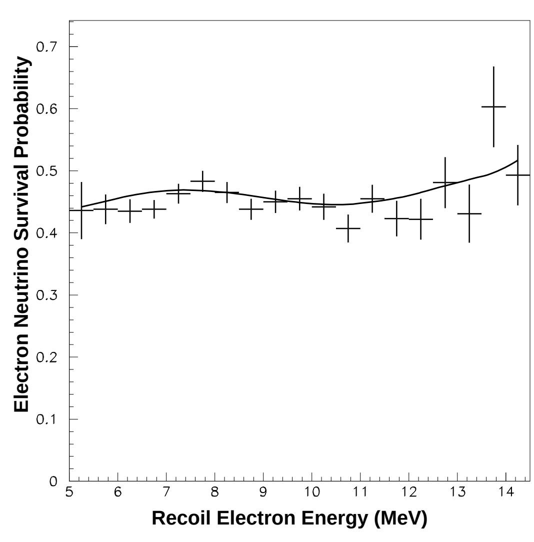

FIG. 2. Super-Kamiokande 1258-day measured [4] energy spectrum (error bars) and predicted (curve) using the parameters of Fig. 1 without fitting these data.

The seasonal effect was computed for a few points on the Earth's orbit. If r is the distance between the Earth and the Sun, $\frac{r_0}{r} = 1 + \epsilon\, cos(\theta - \theta_0)$, where r_0 is one astronomical unit, $\epsilon = 0.0167$ is the orbital eccentricity, and $\theta - \theta_0 \approx 2\pi(t - t_0)$, with t in years and t_0 = January 2, 4h 52m. Table I shows very small seasonal variation.

Not only is the seasonal effect very difficult to observe, but also the smallness of the mixing angle makes day-night effects hard to measure. In addition, the mass of the electron neutrino, which consists mainly of eigenstates of mass 3×10^{-5} eV, is undetectable directly or by neutrinoless double beta decay. The latter process measures an effective neutrino mass, but even contributions to that from the ν_μ and ν_τ must be so small as to make detection very unlikely, although other conjectured processes unrelated to neutrino mass could cause this decay.

On the other hand, the dimension size of 0.06 mm, suggested by the average rates of the three types of solar experiments should be detectable by gravity experiments. The present best limit [17] on such effects is less than a factor of four from that value.

The large size of the extra dimension raises issues about cosmological and supernova limits from the effects of high KK states of both sterile neutrinos [18–20] and gravitons [21], despite uncertainties in the understanding of the complex regimes of the early universe and the supernova core. For sterile neutrino limits, this phenomenology is aided because there is a single KK tower based on an exceedingly small mass, the VO Δm^2 is an order of magnitude smaller than usual, and for MSW the equivalent $\sin^2 2\theta$ is more than an order of magnitude smaller than for standard fits. For the global B-L model, the universe re-heat temperature could be very low (> 0.7 MeV works cosmologically), reducing production of high KK states. The high string scale of the local B-L model would appear to avoid all these constraints [22], however. More complete investigation of these constraints may enable choosing between these quite different models, both of which provide this new way to explain solar, atmospheric, and LSND oscillation data and may give the first evidence for an extra large dimension.

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TABLE I. Predicted seasonal variations in ν_e fluxes, excluding the $1/r^2$ variation. The model assumed $\mu_0 = 0.32 \times 10^{-2}$ eV, $m_0 = 0.34 \times 10^{-4}$ eV, and $\delta_{ee} = 0.78 \times 10^{-7}$ eV.

$\theta - \theta_0$	Chlorine	Gallium	Water
0 (January 2)	0.3787	0.5144	0.4635
$\pm\pi/2$	0.3762	0.5121	0.4633
π (July 4)	0.3747	0.5082	0.4631